

Chapter 10

Mathematical Genesis

Abstract: Bourbaki demonstrated that mathematics can be generated by three “mother structures”, lattice, group and topology. Piaget discovered these same generators in the psychological development of children. I¹ proposed that lattice order, group opposition and topological transformation are embodied as action, information, and material structure at the physical level. They are also the generators of primordial processes at each level of organization.

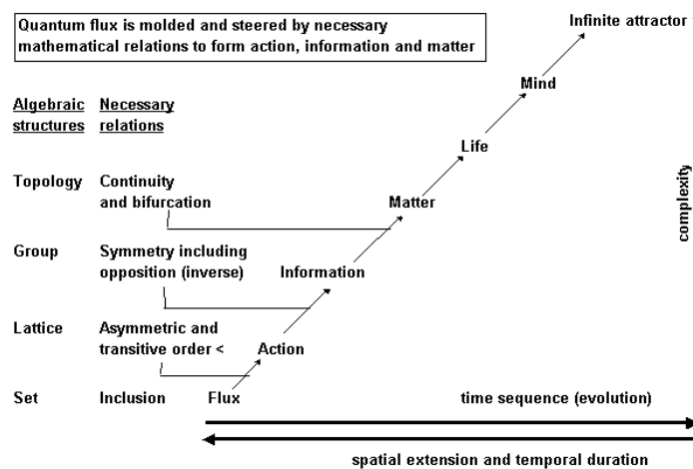


Fig. 10.1 Levels of organization evolve from simple to complex. The simpler levels have larger spatial extension and duration, starting with flux, which is universal, and becoming increasingly more local and transient as they become more complex. The fundamental mathematical structures are defined by simple necessary relations and correspond to simple aspects of physical entities.

¹ Sabelli, H. (1989). *Union of Opposites*. Lawrenceville, VA: Brunswick.

Mathematics is a natural science that describes the most general forms of processes in nature and mind. Mathematical forms constitute the simplest level of organization because their validity does not require the existence of physical entities. Mathematical relations are logically necessary: 2 plus 2 is always 4, regardless. Being necessary, mathematical relations are universal in time and space. Fluxes in the void as well as all other physical entities necessarily embody the fundamental mathematical forms of set, order, opposition, space, and number. It is proposed that the embodiment of mathematical form by the ever-present flux generates reality. Form distinguishes existence from the void. The interaction of groups of actions generates episodic, diverse, novel, and complex patterns. The small integers and irrational numbers such as e , π , and φ are simple universal archetypes; creation involves the generation of higher dimensions. These hypotheses represent a specific formulation of the concept of generic, archetypal numerical and geometric forms advanced by many thinkers from Pythagoras and Galileo to Gödel, Pierce, and Mandelbrot. Archetypes are creative engines. The gene is, in my view, an appropriate metaphor for this concept.² Cosmic forms serve a “boot-strapping” role to create new and complex processes, just as genes serve to develop organisms, and innate psychobiological action serve to generate more complex behavior.

Natural processes and structures display mathematical forms. The logistic equation, fractals, and cellular automata have demonstrated how simple mathematical processes can generate complex and unpredictable outcomes. Biotic recursions show that simple processes involving action, bipolar feedback, and conservation generate creative features such as novelty and complexes. It is thus cogent to ask ourselves how does the mathematical structure of natural processes contribute to their creativity. Such a mathematical model could describe in an abstract manner fundamental processes at all levels of organization.

Prima facie, it seems incongruous to consider the possibility that mathematical structure creates reality. Consider, however, the standard model of particle physics that combines quantum mechanics and special

² Sabelli, H. and Carlson-Sabelli, L. (1996). A cosmic gene? A biological model of complex systems. In honor to James Miller. *Proc. International Systems Society*. 40th meeting, Louisville, Kentucky, July 14-19. Edited by M. L. W. Hall. pp 531-542.

relativity. The model is widely accepted because no significant deviations from it have been observed despite intense experimental testing. The model includes 16 fundamental particles, all of which have been discovered. However, the standard model requires them all to be massless, which is clearly false.³ An additional particle, the Higgs boson is postulated to be responsible for a mechanism by which all other particles acquire mass.⁴ In a similar manner, a mathematical model that describes fundamental processes at an even more abstract level than the standard model of physics, may acquire physical embodiment. I propose that the ever-present quantum flux provides physical substance to mathematical forms.

10.1 Mathematics as a Natural Science

Mathematics is a natural science that describes the most general forms of processes in nature (Galileo) and mind (Piaget). Mathematics deals with reality, postulating axioms believed to be true and confirmed pragmatically by the empirical truth of the theorems derived from them. It distinguishes itself from other natural sciences only in that it obtains very few concepts and relations directly from experience, and infers from them the laws of more complex phenomena by purely deductive means. The idea of securing knowledge by logical deduction from unquestionable principles was explicitly proposed by Aristotle, and successfully applied by Euclid in his *Elements* and later on by Galileo, Newton and their successors. From its inception in ancient Greece, and again in modern times, science adopted a mathematical interpretation of nature.⁵ This is not arbitrary, proposes psychologist and mathematician Robin Robertson.⁶ Einstein wondered how it was possible that mathematics, a product of human thought, so admirably described reality. Mathematical science is “unreasonably effective” in describing

³ Renton, P. (2004). Has the Higgs boson been discovered? *Nature* 428: 141 – 144.

⁴ Higgs, P. W. (1964). Broken symmetries, massless particles and gauge fields. *Phys. Lett.* 12: 132–133; Higgs, P. W. (1966). Spontaneous symmetry breaking without massless bosons. *Phys. Rev.* 145: 1156–1163.

⁵ Randall, Jr., J. H. (1940). *Making of the Modern Mind*. Boston: Houghton Mifflin Co.

⁶ Robertson, R. (1989). The evolution of number. *Psychological Perspectives* 20: 128-141; Robertson, R. (1995). *Jungian Archetypes*. York Beach, ME: Nicholas Hays.

physical reality.⁷ Mathematical science accounts for, and predicts, natural processes; e.g. calculations almost precisely determine interplanetary travel. We can calculate with surprising accuracy. We can demonstrate with certainty properties of numbers, geometric figures, and abstract algebraic structures, suggesting that these products of our minds also exist objectively. Mathematical science must then describe the mathematical forms of nature; it cannot be just a human invention.

Through mathematics, the human mind must undoubtedly come into contact with fundamental physical processes. A very limited period of mathematical development (numbers were invented a few thousand years B.C. at the earliest) has proved sufficient to encapsulate many fundamental laws of nature. Moreover, we can learn this fundamental mathematics in a fraction of our own life span! Undoubtedly our learning abilities must be connected with the fundamental mathematical forms of the cosmos. Mathematical thinking discovers nature. Numbers constitute a universal alphabet. We may speak different languages, hold different beliefs, idealize different heroes, but we all agree when it comes to numbers. Number is a paradigmatic example of the fit between physical reality and psychological processes.

In his 1933 Oxford lecture, Einstein highlighted the physical, empirical nature of mathematics by inviting us to consider Euclidean geometry as the science of the possible mutual relations between practically rigid bodies in space –in other words, to treat geometry as a physical science, without abstracting from its original empirical content.⁸ Without considering geometry as a natural science, Einstein adds, he could have never formulated the theory of relativity. Supporting the view of mathematics as a natural science that describes the logic of nature, mathematics often pre-discovers physical reality. Dirac's equation predicted the positron.⁹

Mathematical forms are objective forms of natural processes and structures; they are not just mathematical concepts developed in our mind. The objective mathematics of the universe is reproduced in the

⁷ Wigner, E. (1960). The Unreasonable Effectiveness of Mathematics in the Natural Sciences. *Comm. Pure Appl. Math.* 13:1.

⁸ Einstein, A. (1954). *Ideas and Opinions*. New York: Wing Books, p. 272.

⁹ Farmelo, G. (2002). *It Must Be Beautiful: Great Equations of Modern Science*. Granta Books.

“subjective” mathematics discovered by human minds. A “mathematical form” describes in an abstract manner patterns, shapes and structures in the universe. Generic forms (“ideas”) have an objective existence as physical properties, shapes, relations, and dimensions.

The fact that natural forms are portrayed by abstract mathematical concepts suggested to Pythagoras and Heraclitus, and later to Galileo and modern scientists, that there is a logic to the universe, a rationality, a *logos*. Human mathematics describes this logic of the universe.

10.2 Mathematical Priority, Psychological Supremacy

We discover mathematical structures in nature. Mathematics starts with intuitions regarding number, space, and so on; these intuitions are uniform in many different people not simply because human beings are all approximately equal but because we all deal with the same external world. Fundamental mathematical endeavors, such as finding the relation π between the circumference and its diameter, display a compelling universality and uniqueness that marks them as discoveries. Arithmetic emerged from accounting, and geometry from surveying and astronomy.

Mathematicians also invent forms that may or may not exist in reality. Do all of them represent forms actually occurring in nature? Or are there mathematical objects created by the human imagination? Some such mathematical inventions undoubtedly are mental constructions – albeit not necessarily only in human minds. The wonderful world of multiple infinite numbers, for instance, seems to me a clear example. The development of non-Euclidean geometries in the 19th century suggested that mathematics consists in the exploration of the logical consequence of axioms regardless of their possible empirical validity.¹⁰

¹⁰ In his *Foundations of Geometry* (1899) David Hilbert considers three arbitrary collections of objects, which he calls ‘points’, ‘straight lines’ and ‘planes’, five undefined relations between them, and 20 axioms which are sufficient to characterize the said objects and relations up to isomorphism. This structural equivalence, however, can hold, between different, intuitively disparate, systems of objects. The formalist school of mathematics led by Hilbert, thus attempted to convert mathematics into a purely formal system in which its objects (numbers, points, sets) were to be defined solely by its axioms such that they were totally empty of content and without reference to the objective world. In contrast, the discoverer of non-Euclidean geometry, Lobačevskii, considered that geometrical truth, like other physical laws, can only be verified by experiment. Surprisingly, the universe appears

The centuries long polemic between Platonists who assume that mathematical laws are discovered in nature and formalists who view mathematics as human invention suggests that both views reflect complementary sides of mathematical science. The concept of priority of the objective and supremacy of the subjective provides a way to understand this dual nature of mathematics: human mathematicians discover natural forms. There are two things that we call mathematics: the mathematical structure of nature that has priority and mathematical science that has supremacy.

There are two aspects to mathematical science: discoveries and inventions. The mathematical structure of physical and biological reality has priority in the formulation of mathematical science, while human imagination has supremacy in constructing mathematical concepts that may or may not correspond to physical reality as well as in deciding which ones we take at this time as true models of reality. The mathematical forms of nature are discovered. They exist before us; they have priority. We also transform them, and create from them new mathematical forms. This does not separate mathematics from other natural sciences. Physics does not cease to be a natural science because accelerators create new chemical elements besides those found in nature.

10.3 Mathematical Certainty and Biological Mediation

Mathematical relations are both necessary and certain. They hold true whether or not there are natural entities that realize and actualize them. Two plus two equals four, with total certainty, in both natural processes and human reasoning. Physical processes cannot depart from the laws of mathematics, at any time or place. Necessary mathematical forms and relations are thus materialized in spatial structures and physical processes, and beyond in brain and mind. Mathematical organization has logical and hence temporal priority over the universe.

Mathematical form is the simplest level of organization because it does not require the existence of physical entities. It also goes beyond

to be Euclidean. This empirical refutation does not diminish the importance of non-Euclidean geometries, showing that mathematics is not only a description of reality.

physics to include biological and psychological processes that in turn create mathematical science.

The priority and universality of mathematics implies that in the physical universe, mathematical certainty is more fundamental than quantum uncertainty. As the necessary form of natural processes, mathematics has logical and temporal priority, informational certainty, and spatial universality.

Mathematical forms arise from our bodily experiences; they abstract human actions, not only external forms. Other rational beings could experience differently from us, and thus develop a different set of “metaphors” which could equally well serve to understand certain aspects of nature. Could these reasoning organisms come up with an entirely different mathematics? Could aliens have invented a different geometry? An experiential view of mathematics has been advanced by the leading Swiss psychologist Jean Piaget.¹¹ According to some Kantian philosophers, number, space and time lack objective reality, being mere categories imposed by our minds.

This philosophical idealism does not follow from fact. Human experiences are objective: they are interactions with natural objects. Mathematical abstraction, as any other scientific concept, involves both object and human agent. The “subjects” are also objective/material/natural, and their actions are largely congruent with objective reality because they are intended to be efficacious. I thus regard the objectivist and the experimentalist views as complementary.

It seems rational to accept that the mathematics of nature are certain, but what provides certainty to a mental process? What makes mathematical science certain? This is not evident. In fact, one should expect that some models would fit reality, however roughly, while others may be quite off the mark. Einstein quibbled that as far as the proposition

¹¹ Chicago mathematician Saunders MacLane also proposes that mathematical ideas are abstract formal representations of human activities, such as counting (arithmetic), shaping (geometry), estimating (probability theory), proving (logic), grouping (set theory), etc. Berkeley cognitive scientist George Lakoff (*Women, Fire, and Dangerous Things*, Chicago: University of Chicago Press, pp. 355-361, 1987) also rejects the “objectivist” view of mathematics and replaces it with an experiential one. This experiential viewpoint seamlessly becomes subjectivism. Jungian psychologist Marie Louise von Franz (Jung, C. G., Franz, M. L., Henderson, J. L., Jacobi, J. and Jaffé, A. (1971). *Man and His Symbols*. Garden City, NY: Doubleday) asserts that in examining nature, “man encounters himself instead of looking for and finding objective qualities” (my underlining).

of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. Human mathematics do not always fit nature. For instance, theories of infinity clash with intuition, and infinite quantities keep appearing in quantum mechanics and need to be removed by *ad hoc* mathematical means. Thus Polish logician Jaroslaw Mrozek¹² proposes that mathematics is neither mathematical nor non-mathematical, but simply amathematical, to explain both the successes and the failures of applications to the natural sciences.

To deprive nature of logic and rationality because we sometimes misinterpret it seems to me an exaggeration. The notion of priority of the objective and supremacy of the subjective resolves the issue of failures in human descriptions of nature. Because all perceptions and concepts are in part generated by our minds, they can be erroneous. But they are for the most part true reconstructions of reality. The explanation is that mind exists in the brain.

Biology accounts for the fact that mental mathematics fits the mathematics of nature. American scientist Larry Vandervert¹³ explains: the human brain portrays the universe realistically because it has developed through evolutionary processes that encapsulate the world as neurological order. Mathematical structure is a most dramatic example of the homology between simple and complex levels of organization. Physical cosmic forms (Platonic ideas) are embodied in the human brain.¹⁴ The three dimensions of macroscopic physical space determine that the labyrinth of the ear has three orthogonal semicircular canals, and this in turn makes us perceive space as tridimensional. Perceptions and mathematical intuitions provide us with a reasonably appropriate, albeit certainly not perfect, picture of reality. While this notion is labeled “naive realism” against which one can raise scientific caution and ingenious philosophical arguments, nobody behaves based on contrary assumptions. Instead of arrogantly dismissing natural perception as possibly flawed, it seems prudent to learn from nature.

¹² Mrozek, J. Did Einstein Claim That Nature Has Mathematical Structure? filjam@univ.gda.pl.

¹³ Vandervert, L. R. (1993). Neurological Positivism's Evolution of Mathematics. *Journal of Mind and Behavior* 14: 278; Vandervert, L. R. (1988). Systems thinking and a proposal for a neurological positivism. *Systems Research* 5: 313-321.

¹⁴ For an opposite view, see Davies, P. C. W. (1990). *Why is the Universe Knowable? Science and Mathematics*, edited by R. Mickens. Singapore: World Scientific.

Numbers must be neurologically coded, no matter how vaguely, because infants¹⁵ as well as animals¹⁶ can differentiate between small numbers, and adults can perfect their use to extremes of precision, abstraction, and creativity. From these modest biological origins, numbers develop as concepts because we encounter numbers in real life. Counting, ordering and measuring are fundamental human actions, in both everyday life and scientific pursuits. Counting with stones (Latin “*calculi*”) lie at the origin of calculus, just as measuring land is the origin of geo-metry. We learn of solids by touching bodies, of flat surfaces by looking at water, of lines by edges. We develop abstract concepts such as “two” through experience, as illustrated by the many different terms used to name concrete twosomes, such as couple, yoke, duo, pair, etc. The laws of algebra are learned unconsciously while performing ordinary arithmetic.¹⁷ Conceptual categories, in part genetically inherited, are subsequently developed as experience confirms, reinforces and elaborates on the simpler inborn patterns. With the progress of physics come new intuitions and hence also new mathematical structures. Intuition is not a mystical process, but a direct apprehension of a natural pattern. As perceptions, intuitions are often fallible, but they are also the source of evidence. The intuition of mathematical relations appears to have double roots in inherited psychological structures and in practical experience. Mathematics is thus capable of revealing truth because its ideal objects and processes aptly model real objects and processes.

10.4 Set Theory

For much of recent history, set theory has been regarded as the foundation of mathematics. Set theory largely originates with a set of articles by Georg Cantor, published between 1879 and 1884, although

¹⁵ Wynn, K. (1998). Psychological foundations of number: numerical competence in human infants. *Trends in Cognitive Sciences* 2: 296303; Xu, F. and Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition* 74(1):B1-B11. For an opposite view, see Feigenson, L., Carey, S., Spelke, E. S. (2002). Infants’ discrimination of number vs. continuous extent. *Cognitive Psychology* 44:33-66.

¹⁶ Brannon, E. M. and H. S. Terrace. (1998). Ordering of the numerosities 1 to 9 by monkeys. *Science*: 746-749.

¹⁷ Spencer-Brown, G. (1969, reprinted 1979). *Laws of Form*. New York: E. P. Dutton.

Bernard Bolzano had already proposed the notion of set as the idea that we conceive when we regard the arrangement of its parts as a matter of indifference. This is contradictory to the notion of process.

It may be possible to develop a different foundation for mathematics from a process perspective. Just as objects are modeled by sets, processes must be modeled by series, which are open ended in time, and hence varying in content. A theory of series is the process equivalent to the logic of sets. We thus view numbers as series, rather than as sets. Thus 0 is a flux, not an emptiness. Likewise, one should consider processes of infinitating but not infinite as an achieved set; infinitations may be regarded as multiple, insofar as they have different initial points –for instance, in bipolar feedback recursions, infinitations have multiple origins and are presumably directed towards different “endings”.

Although set theory is considered as the foundation of mathematics, it is flawed by paradoxes. Cesare Burali-Forti¹⁸ found contradiction in the fact that the ordinal number of the set of all ordinals must be an ordinal. Cantor discovered the paradox implicit in asking what is the cardinal number of the set of all sets. Russell and Zermello independently discovered “Russell’s paradox.” Yet in spite of its paradoxes, most mathematicians accepted set theory as an intuitive foundation for all of mathematics. In my view, the set theory paradoxes point to the need for new concepts. The paradoxes may disappear adopting a process rather than a static perspective in regarding infinity.

10.5 Bourbaki’s Mother Structures of Mathematics

Nicolas Bourbaki reorganized mathematics in a way that exerted considerable influence on researchers and educators all over the world for the ensuing decades.¹⁹ He created a logically ordered totality based

¹⁸ Burali-Forti, C. (1967). A Question on Transfinite Numbers. In *From Frege to Gödel: A Sourcebook in Mathematical Logic, 1897-1931*, edited by J. Heijenoort. Lincoln, NE: toExcel, 104-112.

¹⁹ While adopting the Bourbakian notion of three mother structures, many mathematicians and physicists recommend major departures. The Bourbaki program minimized both the heterogeneity of mathematics and its empirical roots. Many areas of mathematics originate with empirical science. Mathematics as we know it at any given time does not appear to be a hierarchy of increasingly more