

Sabelli, H. and Carlson-Sabelli, L. (1996). As simple as one, two, three. Arithmetic: a simple, powerful, natural and dynamic logic. *Proc. International Systems Society*. pp 543-554.

**As simple as one, two, three.
Arithmetic: a simple, powerful, natural and dynamic logic**

Hector Sabelli and Linnea Carlson-Sabelli
Chicago Center for Creative Development
and Rush University
Chicago, IL, USA.

"The Tao begot one.
One begot two.
Two begot three.
And three begot the ten thousand things.
The ten thousand things carry yin and yang.
They achieve harmony by combining these forces."
Lao Tzu.

Abstract: Numbers describe form and complexity, in addition to quantity and order. In this manner arithmetic can be interpreted as the natural logic of objective processes and of rational thinking.

Key words: asymmetry, complexity, dialectics, dimensions, logic, number, union of opposites, process theory.

Human survival depends on development of a new manner of thinking (Einstein). The power of a method depends on both its intrinsic capabilities and its widespread use. Arithmetic recommends itself on both counts. Here we propose to interpret arithmetic as the natural logic of real processes and of rational thinking. Scientific support for this approach includes the striking correspondence of arithmetic calculation with real processes, the role of numbers as mental archetypes [Jung, 1960] originating from the homology of brain and reality [Vandervert, 1988], the fundamental place of number theory in mathematics (Gauss), the correlation between changes in quantity or intensity with changes in quality [Engels, 1940], and the demonstration that arithmetic cannot be reduced to a logical system simpler than arithmetic itself (Gödel's theorem [Gödel, 1981; Wang, 1988]). Early in the century, it was hoped that science could be unified through its reduction to physics, that physics could in turn be formulated mathematically, and that all mathematics could be accounted for by mathematical logic. An unavoidable consequence of such reduction is a static view, contradictory to natural evolution and social progress. Although the reduction of complex phenomena to simpler processes is a useful scientific strategy, reductionism neglects essential aspects of complex processes. Thus the need for the complementary perspective of the system sciences.

Arithmetic is here interpreted as a general theory of processes and thinking. Natural laws are written as numerical forms in the pattern of processes and the structure of matter. The idea is not new, stemming from Pythagoras and Galileo. Here it is here connected to Gödel's theorem [1981], to modern quantum physics [Capra, 1975; Honig, 1995], and it is developed as a dynamic logic. One may be initially be put off by such an abstract formulation of systems principles, but it provides a method to study complex processes, which is illustrated here and elsewhere [Carlson-Sabelli et al, 1990, 1994, 1995; Sabelli and Carlson-Sabelli, 1995; Sabelli et al, 1995] with empirical data. Further, it places a powerful and dynamic mathematical logic within the reach of persons with grammar school education.

Numbers: cardinal, ordinal, formal: Numbers are interpreted in a cardinal sense, as quantity (one, two, three...) or in an ordinal sense to convey priority (first, second, third...). In addition, numbers may have a third role as descriptions of form, as suggested Pythagoras in antiquity, and by Gödel [1981] and Jung [1960] in our times. Such conceptual expansion of the concept and uses of numbers reflects human history.

In the hierarchical medieval system, order dominated thinking. In modern times, money, travel and measurement has placed quantity at the center of thinking. Thinking in terms of process and system, however, requires also qualitative thinking. This is illustrated by the current shift from numerical calculus to qualitative dynamics (chaos theory, catastrophe theory).

The digital coding of sounds, colors, and other qualities achieved in electronics indicates that at least in principle qualities can be represented by numbers. It is possible that qualities and the classes they define, such as pears, apples, red, virtuous, may be also coded as numerical forms in nature.

Some forms are defined by adimensional numbers, such as Pi and e, that represent circular and exponential patterns. The adimensional Froude, Knudsen, Mach, and Reynolds numbers describe dynamic characteristics of mechanical systems [Parker, 1984]. Numbers also describe psychological processes, such as aesthetic preferences. Pythagoras discovered that harmony in music corresponds to simple numerical ratios in the length of the strings; for instance, if the length of a string is halved, then it sounds one octave higher. Music was for a long time a unique example of numbers in science [Wells, 1986]. Phi (1.61803...) is an asymmetric proportion that is aesthetically pleasing (as illustrated by its use from the Egyptian pyramids, the Parthenon and Renaissance paintings to contemporary post cards and book formats), and hence called "golden ratio" or "divine proportion". Thus Gustav Fechner adopted phi in his attempts to set esthetics on an experimental psychology basis [Huntley, 1970]. Phi also represents biological proportions such as the ratio between branches of the bronchial system [West and Goldberger, 1987]. Phi also is the ratio between successive Fibonacci numbers, a series created by the addition of consecutive numbers ($1+1=2$; $1+2=3$; $2+3=5$; etc...), a series that describes spiral growth, and hence many natural structures and processes [Cook, 1979; Marchetti, 1986].

Dialectics highlighted the association between numbers and qualities. Engels [1940] proposed as a law of both natural and mental processes that changes in quantity produce changes in quality, either gradually or suddenly. For instance, water freezes at 0° ; varying degrees of sweetness are perceived as pleasant or unpleasant; an increase in population changes a village into a city. The qualitative changes associated with quantitative changes indicates the logical sophism implicit in the a potiori arguments (taking an extreme case as a paradigmatic example) often adduced by Freud.

Several numbers can be readily interpreted as qualities: 0 represents non-existence of matter (void), movement (rest) or order (randomness); 1 represents unity, oneness (substance), unidirectionality (time), and universality; 2 represents pairing, complementarity, opposition, difference and information; 3 are the dimensions of space and structure; infinity has likewise a qualitative connotation. Hausdorff dimensions serve to characterize fractal forms [Abraham, 1990].

Time series exemplify the relation between ordinal, cardinal, and formal numbers: a time series is a relation between order and quantity, such as duration or magnitude, which creates a temporal form or pattern, the dimensions of which can be measured. Some time series, such as Fibonacci's, generate simple and evident forms. Including ever present limits to growth generates the logistic equation [May, 1976], the study of which gave its name to chaos [Yorke and Li, 1975]. As the degree of opposition increases, the equation generates 1, 2, 4, 8, 16, outcomes, a cascade of bifurcations that leads to chaos, which is in turn interrupted by simpler periods (3, 5, 7...), each followed by bifurcations and chaos. The existence of period three implies that chaos will be generated, and in turn chaos creates every possible numerical periodicity [Yorke and Li, 1975]. Logistic patterns thus exemplify how opposition creates organization.

Numbers as cosmic forms and as psychological archetypes: Numbers exist in nature. That a flower has five petals is as much part of objective reality as that its color is red (Gödel [Hao Wang, 1988]). Fiveness is a form that exists in the plant's genetic code before and after its physical embodiment in petals. Certain forms occur repetitively in physical, biological and psychological processes. They are thus considered as "cosmic forms". A number of such forms, most notably Pasteur's cosmic asymmetry, are discussed in a companion article in these Proceedings [Sabelli and Carlson-Sabelli, 1996], where the reader will find appropriate references. In this sense we may say that mathematical form (such as number) is more fundamental than material reality. Form is the common ground between the physical world of processes and the mental world of ideas --"idea" in Greek means form.

Numbers (as well as other mathematical structures) are both cosmic forms and psychological archetypes [Robertson, 1989, 1995], meaning recurrent forms of human thought [Jung, 1934]. As a verb, the

term "arche" means both to rule and to initiate, and as a noun it means principle. Thus "arche" denotes a form universally present in natural processes that serves to create them. A scientific principle is a cosmic form that serves as an origin, a force, a process, a generator of change [Sabelli and Carlson-Sabelli, 1996]. Numbers are both physical and mental: we can calculate physical processes in our minds. Psychological archetypes correspond to cosmic forms because the human brain is the best organ (insofar as we know) developed by the evolutionary processes of adaptation and selection. Thus perceptions, intuitions and calculations should be expected to provide us with a reasonably appropriate, albeit certainly not perfect, picture of the real world [Nicolai, 1976, Vandervert, 1988, Sabelli, 1989]. The labyrinth has 3 orthogonal semicircular canals because space has three dimensions, so we correctly perceive space as tridimensional. Likewise we perceive time as flowing because physical and life processes evolve unidirectionally, a fundamental asymmetry to be added to the basic laws of physics.

If numbers describe forms present in natural processes, then arithmetic provides an abstract model for natural processes. Further, one may employ numerical forms as fundamental postulates for logical inferences, and as guidelines for creative thinking.

Cosmic numbers

In search for a formal interpretation of numbers, we shall start by exploring the hypothesis that the small integers 1, 2, and 3, are cosmic forms that exist in all natural systems, at every level of organization, and in every respect, and that they generate the infinite diversity of nature through the creation of time and space series such as the Fibonacci series and the logistic equation. As we shall see, this constitutes a numerical formulation of some basic hypotheses in process theory [Sabelli, 1989; Sabelli and Carlson-Sabelli, 1989].

(1) Oneness: Process theory postulates oneness, a numerical form, as a fundamental feature of reality in every respect: as oneness of composition, as the partition of all processes and structures into units, as the unity (interconnectedness) of the universe, and as unidirectionality of time and asymmetry of structure.

Oneness in diversity: Both matter and spirit are manifestations of the same stuff, action, which is defined in physics as the product of energy x time, i.e. change, energy flowing in time. This is dynamic monism. There is one, and only one universal stuff that makes everything in the universe. The very idea of system implies the notion of unity and plurality: a system is a composite unity of many parts, a diversity within unity. A process is a unity becoming diverse in time. As everything is made of physical energy in constant change, multiplicity and diversity are generated, so the universe is one in composition, and many in organization. This union of the one and the many, the first of the three interlocking theories in Heraclitus' process philosophy [Barnes, 1979; Copleston, 1963], serves as the basis for our interpretation of arithmetic as a model for processes.

By way of contrast, many philosophies and most religions postulate that matter and spirit are two different substances (dualism). Materialism, regarding everything as made of matter, and idealism, proposing that there are only ideas, also postulate oneness, but fail to account for the emergence of diversity.

Unidirectionality and asymmetry: Action is not only uni-versal but also uni-directional. Postulating the asymmetry of time differentiates process theory from mechanics (classic, statistical, relativistic or quantic) that allows for reversibility. Process theory explains the universal asymmetry of structures and processes postulated by Pasteur [Haldane, 1960; Sabelli and Carlson-Sabelli, 1996] as the imprint of the unidirectionality of time. Asymmetry is the defining property of order, the subject of lattice theory, the first pillar of mathematics according to Bourbaki. Perhaps unconsciously recognizing the asymmetry of natural units, we represent oneness with the asymmetric symbol 1, and absence with the symmetric symbol 0. By way of contrast, most theories represent "elements" as symmetric, often structureless and adimensional, units.

Many natural and human processes have asymmetric, long-tailed Pareto-Zipf distributions [Pareto, 1965; Zipf, 1948]. Measuring the entropy of alternating opposites, biological data, chaotic time series, and random numbers [see Carlson-Sabelli et al, 1996 in these proceedings] indicates that biological processes are asymmetric, as contrasted to mathematically generated curves such as alternating opposites, chaotic distributions, and randomness, which are symmetric.

Units: Quarks, protons, atoms, molecules, cells, organisms, communities, nations, planets, solar systems, there are units at every level of organization. Every system is a composite. Every process is discontinuous at some level. Action is quantic (Planck constant), not infinitely divisible, and hence so is energy and time. Quantity is a fundamental consequence of the division of every process into units. **Universe:** The term universe, meaning one flow, reminds us of the unidirectionality of time, and of the interconnectedness of processes highlighted by systemic thinkers from Plato to Laszlo [1995]. The unity of the universe has a limit, as systems also have a degree of local autonomy. Quasi-locality is a fundamental property of the universe [Halliwell, 1990].

The concept of oneness indicates that one must study together the various aspects of a process, rejecting disciplinary barriers as distorting abstractions. The concept of asymmetry and directionality implies to represent states as vectors indicating action, not as static points. Processes thus are represented as trajectories of time series generated by longitudinal recordings; single measurements, no matter how exact, objective, and reliable, are inadequate to portray reality. The units of action as well as the units of uncertainty in measurement, must be identified, and **linear** quantities such as time and energy must be measured.

Oneness of substance, unit of quantity, unity of entity, asymmetry of structure, unidirectionality of action, universality of property, are similar in embodying the same form, one, albeit in different respects. To say that one is a universal form means that it is a trait common to all entities and relations, and hence it has numerous different forms. This same concept applies to every other universal forms, such as twoness.

(2) Twoness: Two is also a universal feature of processes, as represented by information (a difference between two cases) and by oppositions in time (change), space (separation) and quality (difference). Twoness appears in the multiple forms of opposition, such as in pairing, repetition, complementarity and conflict. Unity and diversity, connectedness and autonomy, harmony and conflict, continuity and discontinuity, union and separation, all opposites coexist. A paradigmatic example is quantum complementarity, as an electron is both a particle and a wave. Physical waves, positive and negative electromagnetic polarities, chromosomal pairing, physiological cycles, bipedal walking, and conjugation in aromatic molecules exemplify the alternation between two opposite states. The inseparability of life and death, the coexistence of evolution and decay, the priority of action and the supremacy of organization, are fundamental examples of the coexistence of opposite processes (enantiodromia, from the Greek *enantio*, opposite, and *dromos*, race). The pairing of sexes in reproduction, and of DNA bases in heredity, are good examples of the creative role of complementary oppositions in nature.

Group theory, a second pillar of mathematics according to Bourbaki, studies the relation between opposites (an element and its inverse), and the more complex systems of symmetry they generate. Opposition has been recognized as a fundamental and universal form of natural processes, at least since Anaximander and Pythagoras. The second of the three interlocking theories that constitute Heraclitus' process philosophy is the union of opposites. The union of opposites postulates the universality of opposition and a fundamental commonalities between any two opposites in composition, origin and fundamental properties). The union of opposites can be presented as a numerical form, namely as the co-extensionality of oneness and twoness. There are two (opposites) in every unity, and there is oneness in the unity of opposites. Both oneness and twoness are universal.

The union of opposites is implicit in quantum mechanics complementarity [Capra, 1975; Kothari, 1985; Honig, 1995], in a relativity theory [Honig, 1995]. It is recognized in psychoanalysis (Freud's view of the unconscious, Jung's concept of enantiodromia), and it has been incorporated in process philosophies (Nicholas Cusano, Hegel, Engels), and in system theory [Bertalanffy, 1968; Xu and Li, 1989], albeit partially, postulating that opposites coexist either in harmony or in conflict. The notion of coexistence in both harmony and antagonism is the core of process theory [Sabelli, 1989; Sabelli and Carlson-Sabelli, 1989; Sabelli and Carlson-Sabelli, 1992].

Twoness indicates the need for **non-linear methodology**: (1) Find in each process its fundamental oppositions. Choose, whenever possible, paired opposites such as harmony and conflict, attraction and repulsion as axes for two dimensional plots. (2) Explore the similarity of opposites and the opposition of similars, the synergy of antagonists and the conflict of synergic actions. (3) Plot the values of each opposite separately, and their joint variations in a coordinate plane. Linear processes, as the name indicate, can be represented in one dimension, but cyclic processes require two dimensions. (4) If opposites cannot be

measured directly, consider variation as the carrier of information about opposition (do not neglect it as random error!); plotting rate versus acceleration (phase plane) or return maps creates a two-dimensional trajectory that can be interpreted in terms of the underlying opposites.

Numerical sign, positive versus negative, serves to represent only linear oppositions. Opposites can be represented as extremes of a linear continuum only insofar as changes in quantity correlate with changes in quality (Hegel-Engels), as illustrated by changes in time (child, adult, old), size (village, town, city), or intensity (appetite and hunger). However opposites can increase or decrease independently, and thus have a complex relationship, not just reciprocity. Opposites may have synergic as well as antagonistic components. Only in mechanics opposites neutralize each other; in non-mechanical processes, opposites can coexist in various degrees, and co-create complexity, not equilibrium. One must then represent opposites in orthogonal axes. In this manner each opposite can be decomposed in two vectors, one in which opposites add to each other, and another in which they subtract from each other (provided that the opposites are measured in the same units). These two dimensions determine a plane that can be used as a bidimensional framework (**diamond of opposites**) in which to portray instantaneous states and the trajectory of successive states (**phase plane of opposites**). We use the diamond of opposites (figure 1) as a practical planar scale to measure empirically coexisting opposites (such as ambivalence, contradiction and ambiguity) that coexist in emotion [Sabelli, 1989] and motivation [Carlson-Sabelli et al, 1992]. Categories force black and white distinctions. Linear scales force us to conceive opposite drives or ideas as neutralizing each other, and depict only three cases, positive, negative or neutral. The four quadrants of phase plane of opposites (neutral, positive, negative and contradictory) allow one to portray all cases, and transitions from one to the other. Measuring opposites separately, one can demonstrate that attraction and repulsion towards a choice often are positively related [Carlson-Sabelli et al, 1992]; thus, to consider choices as a continuum along a linear dimension is an error in measurement, as well as a violation of most elementary psychodynamic insight. The contradiction of intense opposites often manifests itself as intense and irrational unilateral choices, followed by sudden changes from one extreme to the other (catastrophe, the simplest bifurcation). Contradictions lead to change, both constructive and destructive, so the detection of contradictions has practical value. The simpler catastrophes are governed by asymmetric and bifurcating parameters which we have shown correspond to the sum (union) and difference (separation) of opposites [Carlson-Sabelli et al, 1992].

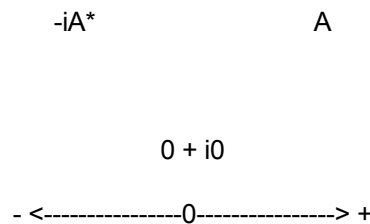
The diamond of opposites acquires further usefulness to interpret return maps and phase plane portraits of time series as well as of entropy measurements (figure 2), based on the relation between coordinates rotated 45°.

Complex numbers as representation of the union of opposites;

A partition A, no-A determines two cases. Repeating negation in another dimension, the i dimension, creates four cases, A, no-A, both A and no-A, and neither A nor no-A, to be represented in a plane. Figure 3 indicates how we can measure opposites in the plane of opposites using complex numbers:

$$A - iA^*$$

Figure 3: The complex plane



Honig [1995] suggested to give a logical interpretation to imaginary numbers to describe the coexistence of opposites in quantum mechanics as well as relativity theory. Whereas sign is adequate only to portray linear opposition, multiplication by i fails to convey the linear component present in all oppositions. Thus negation multiplication by -i seems the most appropriate way to represent opposition. The A axis is

measured by positive real numbers, and the other axis A^* by negative imaginary numbers. Every point in the planar scale is represented by a complex number $A - ia^*$, where i is the square root of -1 , A and A^* are quantities, and opposition is represented by both difference in sign and multiplication by i . Multiplication by i portrays a change in direction and hence an increase in dimension. The change in sign serves to represent the quantitative differences underlying opposition.

This type of representation is widely applicable, as every concept can be defined by the combination of two or more other concepts, that overlap with each other in some respect, and differ from each other in some respect. Consider for instance Plato's definition of man as a "featherless biped": featherless and biped overlap insofar as they are attributes applicable to animals, and diverge insofar as they refer to entirely different characteristics. We may thus represent a concept as an ordered pair, as a complex number, in which the real and imaginary parts represent its conceptual roots, the concepts that generate it when combined.

Traditional thinking would place A and $\text{no-}A$ as the positive and negative poles of the real axis, and consider "both A and $\text{no-}A$ ", and "neither A nor $\text{no-}A$ " as poles of the imaginary axis. A process formulation places "both A and $\text{no-}A$ ", and "neither A nor $\text{no-}A$ " as poles of the real axis, and consider " A " and " $\text{no-}A$ " as pure abstractions to be plotted in the imaginary axis.

2ⁿ. Tetradicity and higher order oppositions: Quaternary is an important psychological archetype [Jung, 1942] as illustrated by cultural constructs such as the four cardinal points, the four seasons, the four elements and the four humors of the ancient Greeks, the four Gospelists, and many four-fold symbols of psychological wholeness described by Jung.

Traditional logic attempts to reduce all questions to two values. However, twoness is manifested in all dimensions. Two oppositions generate four cases, which must all be considered, rather than be reduced to two. In this same manner we can portray to what extent oppositions such as good and evil, beautiful and ugly, true and false, overlap, and to what extent they do not. Good is more beautiful than evil, but obviously also evil can be beautiful, and tempting. Likewise, as every entity has at least two aspects, every relation between entities has 4 aspects. For instance, given that each person has a harmonic and a conflictual side, his relation with another has 4 possible components; harmonic-harmonic, conflictual-conflictual, harmonic-conflictual, conflictual-harmonic, albeit in general each behavior tends to elicit similar behavior in the other. There actually are 16 components, as a given person may be neither harmonic nor conflictual, and can also be both. This multiplication of oppositions—a necessary consequence of the universality of opposition—creates four, eight, sixteen, 2^n cases, resembles a cascade of bifurcations, a common road to chaos [Abraham, 1990], as illustrated by the logistic model of development (see later).

Twoness in process logic: Computers and many other digital devices represent every number and form as a sequence of 0s and 1s. Likewise logic traditionally postulates two values, true and false, and excludes any third case, such as unknown (e.g. the future, as already included by Aristotle in his logic), fuzzy (such as "somewhat tall"), neither one nor the other, contradictory (both true and false, and in dialectics), partly one and partly the other (as in probabilistic logic). As every number can be represented by a sequence of 0s and 1s, the last case can readily be represented by two-valued logic. The repetition of two-valued opposition creates a four-valued logic that includes two cases excluded by strictly two-valued logic, "neither nor" and "both and". As oppositions repeat, we have proposed [Sabelli, 1984, 1995] that the iteration of negation logically implies the existence of a four-valued dialectics. To reduce all possible truth values to the two initial ones is, in our view, the same kind of reductionism at all cost that, in this case, did cost logic to turn-of-the-century logicians.

Opposites coexist (dialectic principle of contradiction, as opposed to the absolute principle of no contradiction of standard logic) but always separated in some respect, such as time, space or dimension (Aristotle's local principle of no contradiction). This provides a logic compatible with the quantum principle of complementarity and with space-time relativity. Being globally contradictory and locally non-contradictory, quantum complementarity represents the paradigm of the process view of opposition as the coexistence of opposites (dialectic contradiction) but separated in some respect, such as time, space or dimension (Aristotle's principle of local no contradiction). Asymmetry at each time, in each place, and in each respect, implies local no-contradiction.

Within a process perspective, we propose to postulate the global coexistence and local exclusion of opposites: p if and only if no-p (principle of the mutual implication of opposites), and p and no-p only if p is true in one regard and no-p is true with reference to a different relation. In this context, complementarity is a dialectic tautology.

Traditional logic postulates either A or no-A; a third case is excluded. Process logic postulates **either A and no-A or neither A nor no-A**; neither A, nor no-A, exist separate from their opposite. Further, process logic postulates the universal existence of a third corresponding to each opposition.

(3) Trifurcation and triunion: Threeness is also a universal form of nature and thought. Physical space has three dimensions. Processes have three inseparable components, energy, information, and matter. Ordinary matter is made of three components, quarks, leptons and bosons. Three quarks assemble to make protons and other hadrons, following a logic analogous to that of visual colors (quantum chromodynamics). Color vision arises in the retina as a three-way split of light frequencies. (The remarkable logical properties of colors are discussed in a companion article [Sabelli and Carlson-Sabelli, 1996]). DNA hereditary information is coded by triplets. Mother, father and child make the nuclear family. Empirical factor analyses consistently indicate three dimensions of variation of normal personality [Cloninger, 1987]; and many models of psychological processes (such as Freud's id / ego / super-ego, Berne's child, adult and parental ego-states, Peirce's I, Thou and It) are tripartite. Modern constitutions provide for a tripartite division of executive, legislative and judicial powers. Many religions propose a trinitarian concept of God; in Greek mythology there were 3 Fates, 3 Furies, 3 Graces, and Paris had to choose between 3 Goddesses. There were three Wise men, three musketeers, and three stooges. The conjugation of verbs includes three persons (I, you, them) and three basic tenses (past, present and future). There are three fundamental types of reasoning, induction, deduction and abduction [Eco and Sebeok, 1984]. Classification by division into three parts is extremely common. Philosophical categories have been perceived as organized in triads by Kant, Schilling, Hegel, Peirce, etc. Many laws of nature, such as Coulomb's $V = I \cdot R$ and Einstein's $E = m \cdot c^2$ relate three variables. A bifurcation differentiates three entities, one precursor and 2 successors), etc. Catastrophes and chaotic attractors require a tridimensional space. Period three implies chaos, assert Yorke and Li [1975] in the article that gave its name to chaos science, as they observe that in the logistic equation (a simple model for growth checked by opposition) chaotic solutions gave way to a set of three alternative outcomes that in turn generated new and wider chaos. Tridimensional thinking promotes creativity [Torre's triadic theory, 1995], whereas white-and-black thinking characterizes ideological fundamentalism as well as psychopathology (neuroses, depression, borderline). We thus propose that **trifurcation** (three-way split) and **triformation / triunion** (three-way convergence) are fundamental processes that create novelty and complexity at every level of organization.

Fractal organization: the iteration of 1, 2, 3: The forms 1, 2 and 3 occur in, 2 and 3 dimensions. For instance, oneness in 1 dimension is unidirectionality (cosmic asymmetry), in 2 dimensions is the unity of opposites (their inseparability, similarity and common origin), and in 3 dimensions is the spatial structure of every unit. Likewise twoness in 1 dimension is the bipolarity of energy, in 2 dimensions is the orthogonality of different oppositions (determining 4 quadrants), and in 3 dimensions corresponds to the eight-shaped form of a p atomic orbital. A triad is a unit as a quantity, a 2-dimensional triangle in the plane, and a trifurcation in physical space. Repeating 1, 2, and 3 creates a unit:

1 2 3
2 4 6
3 6 9

that itself is iterated. Four represents the cross-opposition, the repetition of the opposition. Six is abundant and basic, if not universal, as illustrated by hexagons in honeycombs and benzene rings, and as a form of regular tiling.

As the same numerical forms repeat at every level of organization, processes have a self-similar structure characteristic of fractals. Just as each number contains within itself all the previous numbers, numerical forms are nested (topologically embedded) in each other (albeit one or another can predominate locally as an attractor). For instance, action is a unity but has two dimensions, energy and time, as well as 3 spatial dimensions. Each unit of information is an asymmetry in the opposition between 2 values, and also

occurs in tridimensional space. A structure consists of three asymmetries, and hence of three bipolarities.

(0) Chaos and the exclusion of absolute zero: Existence and absence are not mutually exclusive opposites. "Nothing" contains, and is contained in everything that exists. Matter is contained in space, and contains more empty space than particles. Information always contains uncertainty. Order contains, is surrounded by, and can generate disorder.

The absence of order, namely chaos, is pervasive. At all levels of organization, processes contain, and are contained, in chaotic flux, which may approach, but never attain, zero. Quantum flux, heat (molecular agitation), turbulence, Brownian motion, noise, chance, deterministic chaotic attractors, Prigogine's creative chemical chaos, and infinite randomness, all exemplify the ubiquity of constant, complex and overtly disorganized flux, of change without a clearly directed action. This is the earliest and most general meaning of chaos (although recently the term has been used mainly to refer to deterministic chaos). Ordered processes contain, and are contained, in disordered flux. Energy and matter exist within, and contain, space in quantum flux. There is a minimum of uncertainty in every process, beginning with the Planck constant, the minimum of action, within which virtual particles can be spontaneously created and destroyed, a real physical process that has measurable consequences. The amplification of the unavoidable uncertainty creates unpredictability in chaotic processes highly sensitive to initial conditions.

Chaos is disorder, but it does not imply zero order; for instance, deterministic chaos, which appears random, can be generated by simple equations. Likewise there is no absolute void, as the vacuum is filled with 3°K background radiation, and is in constant flux, spontaneously generating pairs of opposite particles within the limits of the Planck constant. There is no absolute zero temperature (Nernst's theorem, also known as the third law of thermodynamics), and there is no absolute rest. There is no absolute black; even the blind eye sees lights, and even black holes irradiate energy and information. In logic, the empty set has no members. The dimension p of x is zero if and only if p cannot be predicated of x , e.g. we cannot speak of kind triangles or hairy snowflakes. But even in this case we speak of zero only in one dimension. Nothing is zero in all dimensions. For all x , $x > 0$.

Lack of difference implies uniformity, not formlessness (0 form). The minimum of form is the infinite random, random in infinite dimensions and infinite in extension, lacking any boundary. In reality there is no absolute randomness, only approximations, from quantum flux to heat to deterministic chaos to entropic disorder. Each of these approximations to randomness includes, and is included between, regular, determining periodicities. Thus neither order nor indeterminacy, neither cause nor chance, neither number nor chaos, is more fundamental, but both alternate at each level of organization. When considering a particular level of organization, there is a temporal and hierarchical relation between order and chaos; for instance, random genetic mutations have priority over natural selection, and conversely selection has supremacy over mutational variation.

Recapitulating, the negative cannot be separated from the positive, nothing cannot be separated from something, and there is no absolute zero because uniformity, uncertainty and chaos are ever present. We thus propose the number U (representing uniformity and uncertainty, which is always greater than 0). At the physical level, $U = \text{Planck constant}$. In every measurement, $U = 1 / \text{precision of the measurement made}$ (for instance, $U = 8 \text{ msec}$ in our study of the entropy of cardiac beat-to-beat intervals presented at this meeting [Carlson-Sabelli et al, 1996] because voltage was sampled every $1 / 128\text{th}$ of the second).

The Big Bang of numbers: A Big Bang of numbers is implied by a Big Bang of physical entities. If numbers are primal shapes, and number series may represent modes of development that repeat in natural processes as a theme and variations in music. The same organ, a leaf, varies to form all the parts of a plant. Likewise variations of the same numerical forms appear at all levels of organization, so the study of numbers provides a general morphology applicable to all fields of scientific inquiry. The idea is as old as the Chinese concept of the logos, the Tao, which explains the origin of multiplicity ("the ten thousand things") as resulting from a cascade of bifurcations, and embodying united opposites, the yin and the yang [Lao Tzu].

Hundreds of numerical series, and hence an even larger number of numerical forms, can emerge from the small integers. In fact 0 and 1 are sufficient to generate all the numbers. Among these numerical forms, some have particular interest because they are widely represented in nature. Five is a fundamental botanical and anatomical form (starfish, human hand), and the number of the regular Platonic solids. Eight is

also basic, as illustrated by the 8 logical functions, the musical octave, and the group of eight in elementary physics. Of course 0,1,2,3,5,8 represent the beginning of the Fibonacci series; in this light, the union of opposites may be the first of a larger sequence that includes the coexistence of unity, opposition, triadity, cinquicity, octavicity, etc [Sabelli, 1989]. Complexity and uniqueness may emerge as time series of numerical forms, from the universal forms of (0) disordered flux, (1) uni-directional flow of energy in time, (2) bidimensional opposition and (3) tridimensional structuring, in a universal, iterative, and determined, but eminently creative process of development. That period three implies chaos suggest to us that the coexistence of the small integers as cosmic forms constitutes a cosmic generator of complexity [Sabelli and Carlson-Sabelli, 1996].

We view the interaction of opposites (**co-creation**) as the core of creativity. The interaction of opposites creates stability and pattern. Sexual intercourse procreates new individuals. The binding of oppositely charged particles creates atoms. Resistance slows down action, leading to equilibrium points (static attractor) or to cycles (periodic attractors). Catastrophic folds occur when opposite forces are both strong and of similar intensity [Sabelli, 1992].

A logistic model for the logoi: As illustrated by the simple logistic equation $x_{n+1} = \mu x_n (1 - x_n)$, where $0 < x < 1$, and $0 < \mu < 4$, the interaction of growth (iterations of the equation) with opposition that limits growth, generates cascades of bifurcations, chaotic and periodic patterns, and then becomes unbound towards infinity. Each bifurcation occurs at a specific value of opposition (μ in figure 4); changes in quantity create changes in quality (Engels) and hence information. New and more diverse forms are created. Individuals become unique, as already evident in snow flakes. Cascades of bifurcations provide a model for development in which each new stage represents a differentiation into a new set of opposites, the interaction of which contributes to all subsequent processes. Partition differentiates a formless unity into opposites, and thereby adds a new quality or dimension. This model, which we have applied to psychological development [Sabelli, 1989; Sabelli et al, in press] contrasts with the dialectic model (e.g. Erikson's [1968]) according to which each stage of development represents the resolution of a contradiction of opposites. The logistic equation, in which iteration and opposition generates repetitive bifurcations, chaos, and periodicities [May, 1976], illustrate how numerical forms could function as a relative simple generator that spawns determined development, elicits chaos, and creates novel complexity and diversification.

Simplicity and complexity as numerical form: In the logistic equation, repeated bifurcations create periodic and fractal structures: chaotic complexity and periodic simplicity alternate, and create each other (figure 4). In the same manner, one observes in nature that increased complexity collapses into simpler forms [Cohen and Steward, 1994]. Simple patterns of organization such as units, oppositions and triads reoccur at various levels of organization. Thus the re-occurrence of small integers.

Numerical forms provide standards to create a scale of complexity. Twoness, the alternation of opposites represents the minimum of complexity, and rectangular random distributions offer the upper limit, as illustrated by measurements of complexity as entropy [Carlson-Sabelli et al, 1996] and as median embedding dimension [Carlson-Sabelli et al, 1995; Sabelli et al, 1995].

We conceive the evolution from simple to complex as a process of dimensiogenesis. Non-linear interactions create new dimensions. A tridimensional catastrophe is created by the non-linear competition between opposite point attractors. As every process includes at least two opposite components, every pattern has at least 3 dimensions. More complex forms of organization (molecules, organisms, societies and psychological individuals) represent the creation of higher dimensions. The higher dimensions of biological and psychological processes are as yet unidentified, but their number can be estimated in psychobiological data with entropy and recurrence analysis [Carlson-Sabelli et al, 1996].

In summary, we propose, as hypotheses, that **U, 1, 2, and 3 are universal forms that coexist in all natural processes, and that together are sufficient to generate the infinite variety of reality.** The small integers represent universal forms of natural processes; that the following members of the Fibonacci series represent common, albeit perhaps not universal, natural forms; and that higher numerical dimensions are fundamental features of emergent, more complex forms of organization. We propose the hypothesis that each new level of organization starts with the repetition of the simple and universal principles represented by the small integers, and is characterized by new and higher dimensions. A companion article [Sabelli and Carlson-Sabelli, These Proceedings] portrays such a cosmic generator as a topological form. Since

arithmetic cannot be reduced to logic, we propose to elevate logic to arithmetic. Arithmetical formulation may provide us with a dynamic, rigor, and easy logic.

Acknowledgements: We are thankful to the Society for the Advancement of Clinical Philosophy, to Mrs. Margaret Trobaugh, and to Mrs. Maria McCormick, for their invaluable support for this research.

References

- Abraham, F. 1990 *A Visual Introduction to Dynamical Systems Theory for Psychology*. Santa Cruz, CA: Aerial Press.
- Barnes, J. 1979, *The Presocratic Philosophers*, Vol. 1. London: Routledge and Kegan Paul.
- Bertalanffy, L.O von 1968, *General Systems Theory*, New York: George Brazillier.
- Capra, F., 1975, *The Tao of Physics*. Boulder: Shambhala.
- Carlson-Sabelli, L., Sabelli H.C., and Hale A., 1994. Sociometry and sociodynamics. In *Psychodrama Since Moreno: Innovations in Theory and Practice*. (Karp, Watson and Holmes, eds.) Tavistock, New York/London, pp 146-185.
- Carlson-Sabelli, L., Sabelli, H.C., Hein, N., and Javaid, J., 1990, "Psychogeometry: The Dynamics of Behavior." In *Proceedings of the International Society for the Systems Sciences*. (B.A. Banathy and B.A. Banathy, eds.) ISSS, Portland, OR, pp. 769-775.
- Carlson-Sabelli L., Sabelli H.C., Patel M., Holm K. 1992. "The Union of Opposites in Sociometry: An Empirical Application of Process Theory." *The Journal of Group Psychotherapy, Psychodrama and Sociometry* **Vol 44**: 147-171.
- Carlson-Sabelli L., Sabelli H.C., Patel, M., Messer, J., Zbilut, J., Sugerman, A., Walthall K., Tom, C. and Zdanovics, O. 1995, "Electrocardiography. Illustrating the Application of Process Methods to Comprehensive Patient Evaluation. Complexity and Chaos in Nursing **Vol. 2**: 16-24.
- Carlson-Sabelli, L., Sabelli, H.C., Messer, J., Patel, M., Sugerman, A., Luecht, R. and Walthall, K. 1996, "Cardiac Entropy and Symmetry Are Decreased in Coronary Artery Disease: Clinical Significance and Theoretical Implications Regarding Entropy as Symmetry, Not Disorder." *These Proceedings*.
- Cloninger, R.D. 1987, "A Systematic Method for Clinical Description and Classification of Personality Variants. *Archives of General Psychiatry* **Vol. 44**: 573-588.
- Cook, T. A. 1979, *The Curves of Life*. New York: Dover Press
- Cohen, J. and Stewart, I. 1994 *The Collapse of Chaos* New York; Penguin.
- Copleston, F. 1963, *History of Philosophy*. Garden City, New York: Doubleday.
- Eco H. and Sebeok, T.A. 1984, *The Sign of Three*. Bloomington, Indiana: Indiana University Press
- Engels, F., 1940, *Dialectics of Nature*. New York: International Publishers.
- Erickson, E.H. 1968, *Identity: Youth and Crisis*, New York: W.W. Norton.
- Gödel, K., 1981, *Obras Completas*. Madrid, Spain: Alianza Universidad.
- Haldane, J.B.S. 1960, "Pasteur and Cosmic Asymmetry. *Nature* **Vol. 185**: 87
- Halliwell, J.J. 1990, "Information Dissipation in Quantum Cosmology and the Emergence of Classical Spacetime." In *Complexity, Entropy, and the Physics of Information*. (W.H. Zurek, ed.) Addison-Wesley Publishing Co. pp 459- 469.
- Honig, W.M., 1995, *Nonstandard Logics and Nonstandard Metrics in Physics*. Riveredge, N.J.:World Scientific Press.
- Huntley, H.E., 1970, *The Divine Proportion: A Study in Mathematical Beauty*. London: Dover Publications.
- Jung, C.G. 1934, *The Archetypes and the Collective Unconscious* In *Collected Works*, **Vol 9**. (G. Adler and R.F. Hull, eds.) Bollingen Series XX Princeton University Press, Princeton, NJ.
- Jung, C.G. 1942, "A Psychological Approach to the Dogma of the Trinity." In *Collected Works*, **Vol. 11**: 167.
- Jung, C.G. 1960, "Synchronicity: An Acausal, Connecting Principle" in *C. Jung Collected Works Volume 8. The Structure and Dynamics of the Psyche*. Princeton University Press, Princeton, N.J.
- Lao Tsu. 1972, *Tao Te Ching*. Translated by Gia-Fu Feng and Jane English. New York: Random House.
- Laszlo, E. 1995, *The Interconnected Universe*. London: World Scientific.
- Kothari, D. S. 1985, "The Complementarity Principle and Eastern Philosophy." In *Niels Bohr, A Centenary*

- Volume. (A.P. French and P.J. Kennedy, eds.) Harvard University Press, p. 325.
- Marchetti, C. 1986, "Stable rules in Social behavior." IBM Conference, Brazilian Academy of Sciences May, R.M. 1976, "Simple Mathematical Models With Very Complicated Dynamics." *Nature* **Vol. 261**: 459-467.
- Nicolai, G.F. 1976, *La Miseria de la Dialéctica*, Madrid: Editorial Aguilera
- Pareto, V. 1965 reprinted from 1897 *Cours d'économie politique*. Reprinted as a volume of *Oeuvres Complètes*, Geneva: Droz.
- Parker, S.P. (ed.) 1985, *Encyclopedia of Science and Technology*. McGraw-Hill Book Co., New York.
- Robertson, R. 1989, The evolution of number. *Psychological Perspectives* **Vol. 20**: 128-141.
- Robertson, R. 1995, *Jungian Archetypes: Jung, Gödel and the History of Archetypes*. York Beach ME: Nicholas-Hays.
- Sabelli, H.C. 1984, "Mathematical Dialectics, Scientific Logic and the Psychoanalysis of Thinking" In *Hegel and the Sciences*, (R.S. Cohen and M.W. Wartofsky eds.) D. Reidel Publishing Co., New York: 349-359.
- Sabelli, H.C. 1989, *Union of Opposites: A Comprehensive Theory of Natural and Human Processes*. Lawrenceville, VA: Brunswick.
- Sabelli, H.C. 1995, "Non-Linear Dynamics as a Dialectic Logic." In *Proceedings of the International Society for the System Sciences* (B. Bergvall-Kareborn, ed.) ISSS, Amsterdam, pp.101-112.
- Sabelli, H.C. and Carlson-Sabelli, L. 1989, "Biological Priority and Psychological Supremacy, A New Integrative Paradigm Derived From Process Theory. *American Journal Psychiatry* **Vol. 146**: 1541-1551.
- Sabelli, H.C. and Carlson-Sabelli L. 1992, "Process Theory: Energy, Information and Structure in the Phase Space of Opposites." *Proc of the International Society for the Systems Sciences* (L. Peeno, ed.) ISSS, Denver, pp 658-667.
- Sabelli, H.C. and Carlson-Sabelli, L.C. 1996, "A Cosmic Gene? A Biological Model of Complex Systems. *These Proceedings*.
- Sabelli H.C., Carlson-Sabelli L, Patel M, Levy W. , 1995, "Anger, Fear, Depression and Crime. Physiological and Psychological Studies Using the Process Method. In *Chaos Theory in Psychology and the Life Sciences*. (R. Robertson and A. Combs, eds.) Lawrence Erlbaum Associates, Mahwah, N.J., pp. 65-88.
- Sabelli, H.C., Carlson-Sabelli, L., Patel, M. and Sugerman, A. In press, "Dynamics and Psychodynamics. *Process Foundations of Psychology*." *Journal of Mind and Behavior Special Issue Nonlinear Dynamics and Quantum Theory in Psychology*. (L. Vandervert, ed.)
- Sabelli H.C., Carlson-Sabelli L, Patel M, Zbilut J, Messer J, and Walthall K. 1995, "Psychological Portraits and Psycho-cardiological Patterns in Phase Space. In *Chaos Theory in Psychology* (F. Abraham and A. Gilgen, eds.) Praeger, Westport, CT, pp. 107-125.
- Sabelli, H.C., Patel, M. Carlson Sabelli, L., Sugerman A. and Messer, J. "Entropy as Diversity and Organization in Living Systems. In *Proceedings of the International Systems Society*. (B. Bergvall-Kareborn, ed.) ISSS, Amsterdam, pp. 113-124.
- Sabelli, L.C. 1992. *Measuring co-existing opposites: A methodological exploration*. Doctoral dissertation, University of Illinois at Chicago. Available from UMI Dissertation Services, 1-800-521-0600 Ext 879 (1992).
- Torre, C. 1995, "Chaos in the Triadic Theory of Psychological Competence in the Academic setting." In *Chaos Theory in Psychology* (F. Abraham and A. Gilgen, eds.) Praeger, Westport, CT: 279-294.
- Vandervert, L.R. 1988, "Systems Thinking and a Proposal for a Neurological Positivism," *Systems Research* **Vol. 5**: 313-321.
- Wang, H., 1988, *Reflections on Kurt Gödel*. Cambridge: MIT Press.
- Wells, D. 1986, *The Penguin Dictionary of Curious and Interesting Numbers*. Harmondsworth, Middlesex, England, Penguin Press.
- West, B.J. and Goldberger, A.L., 1987, "Physiology in Fractal Dimensions," *American Scientist* **Vol. 75**: 354-365.
- Xu, L. D., and Li, L. X. 1989, "Complementary Opposition as a Systems Concept." *Systems*

Research **Vol. 6**: pp 91-101.

Yorke, J.A. and Tien-Yen Li 1975, "Period Three Implies Chaos. American Mathematical Monthly
Vol. 18: 985-992.

Zipf, G. K. 1948 Human Behavior and the Principle of Least Effort. Cambridge Mass: Addison Wesley.

Figure 1. Diamond of opposites in family therapy: Left: attraction and repulsion toward significant others are plotted separately in a bidimensional plane. Right: replotting the data by calculating the sum and difference of opposite feelings creates the bifurcating and asymmetric parameters that define a catastrophe (non-linearity in a third dimension).

Figure 2: Bidimensional representation of heart rate: Left: phase plane. Right: return map.

Figure 4: Logistic equation: Value of x after 1000 iterations as a function of changes in the resistance factor μ . Note how a cascade of bifurcations (2, 4, etc) leads to chaos, and the subsequent simplification of chaos to periodicities (3, 5, etc).